Sector Concentration in Loan Portfolios and Economic Capital

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This version: August 2006

Abstract

The purpose of this paper is to measure the potential impact of business-sector concentration on economic capital for loan portfolios and to explore a tractable model for its measurement. The empirical part evaluates the increase in economic capital in a multi-factor asset value model for portfolios with increasing sector concentration. The sector composition is based on credit information from the German central credit register. Finding that business sector concentration can substantially increase economic capital, the theoretical part of the paper explores whether this risk can be measured by a tractable model that avoids Monte Carlo simulations. We analyze a simplified version of the analytic value-at-risk approximation developed by Pykhtin (2004), which only requires risk parameters on a sector level. Sensitivity analyses with various input parameters show that the model performs well in approximating economic capital for portfolios which are homogeneous on a sector level in terms of PD and exposure size. Furthermore, we explore the robustness of our results for portfolios which are heterogeneous in terms of these two characteristics. We find that low granularity c.p. causes our model to underestimate economic capital, whereas heterogeneity in individual PDs causes overestimation. Indicative results imply that in typical credit portfolios, PD heterogeneity will at least compensate for the granularity effect. This suggests that the analytic approximations estimate economic capital reasonably well and/or err on the conservative side.

Keywords: sector concentration risk, economic capital

JEL classification: G18, G21, C1

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For their comments we thank, Marc De Ceuster, Michael Gordy, Thilo Liebig, Janet Mitchell, Joël Petey, Peter Raupach, Andrea Resti, participants in the Research Task Force of the Basel Committee on Banking Supervision as well as participants at the 2006 AFSE conference on 'Recent Developments in Financial Economics' and at the 2006 European Banking Symposium. We thank Eva Lütkebohmert, Christian Schmieder and Björn Wehlert for their invaluable support in compiling the German credit register data and Julien Demunyck and Jesus Saurina for providing us with data from the French and the Spanish credit registers.

The views expressed here are our own and do not necessarily reflect those of the Deutsche Bundesbank or the National Bank of Belgium.

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1. Introduction

Although the failure to recognize diversification within banks' credit portfolios was a key criticism of the 1988 Basel Accord, the minimum regulatory capital requirements (Pillar 1) of the Basel Framework of June 2004 are based on a single-factor model[3] which still does not account for differences in diversification. However, recognizing that banks' portfolios can exhibit credit risk concentrations, Basel II stipulates that this risk be addressed in the supervisory review process (Pillar 2), thus creating a need for an appropriate methodology to measure this risk.

Concentration risk in banks' credit portfolios arises either from an excessive exposure to certain names (often referred to as *name concentration* or *granularity*) or from an excessive exposure to a single sector or to several highly correlated sectors (i.e. *sector concentration*). In the past, financial regulation and previous research have focused mainly on the first aspect of concentration risk.[4] Therefore, in this paper our focus is on sector concentration risk, although granularity is also analyzed. Sectors are defined in the following as business sectors. Although sectors can also be defined by geographical regions, this case is not considered in this paper.

The critical role credit risk concentration has played in past bank failures has been documented in the literature.[5] Therefore, the importance of prudently managing sectoral concentration risk in banks' credit portfolios is generally well recognized. However, existing literature does not provide much guidance on how to measure sectoral concentration risk. Consequently, whether particular levels of concentration need to be translated into an additional capital buffer remains an open question.

This paper contributes to the literature in the following ways. First, we measure economic capital in a CreditMetrics-type multi-factor model and evaluate how important the increase in economic capital is in a sequence of portfolios with increasing sector concentration. The analysis is based on

[3] See Gordy (2003).

[4] See EU Directive 93/6/EEC, Joint Forum (1993) and Gordy (2003).

[5] See, for example, BCBS (2004a).

portfolios which were constructed from German central credit register data. They reflect the average business-sector distribution of the banking system as well as higher sector concentrations observed in individual banks. Information on business-sector concentration of banks is not publicly available, thus central credit registers represent unique sources of data on sector concentrations in existing banks. Our emphasis on empirically observable sector concentrations is therefore an important contribution.

Second, we evaluate the accuracy of a multi-factor adjustment proposed in Pykhtin (2004), which offers a tractable, closed-form solution for value-at-risk (VaR) and economic capital (*EC*) and, thereby, for the measurement of concentration risk. *EC* is defined as the difference between the VaR and the expected loss of a credit portfolio. We have applied a simplified version of the model in order to reduce the computational burden. Such a methodology could be useful for risk managers and supervisors in search of robust, fit-for-purpose tools to measure sector concentration in a bank's loan portfolio.

In the empirical part of this paper we show that economic capital can substantially increase with sector concentration across portfolios. Its increase from a credit portfolio representing the average sector distribution of the German banking system to a portfolio that is concentrated in a single sector can be as high as 50%.

In the theoretical part we explore whether sector concentration can be approximated by a simplified version of the model in Pykhtin (2004), which offers a tractable method to avoid Monte Carlo simulations. The methodological framework of the Pykhtin model builds on earlier work by Gordy (2003) and Wilde (2001) on granularity adjustments in the asymptotic single risk factor (ASRF) model. Whereas the granularity adjustment deals with an unbalanced exposure distribution across names, the Pykhtin model offers a treatment for an unbalanced distribution across (correlated) sectors. The *EC* is given in closed form as the sum of the *EC* in a single risk factor model (in which the correlation with the single systematic risk factor depends on the sector) and a multi-factor adjustment term. The model allows banks and supervisors to approximate economic capital for loan portfolios without running computationally intensive Monte Carlo simulations. Furthermore, it poses only moderate data requirements since it requires the input parameters exposure size and probability of default (PD) only on a sector level.

We found that for portfolios with highly granular and homogeneous sectors, the analytic approximation formulae perform extremely well. Moreover, the multi-factor adjustment term is relatively small, so that *EC* in the single risk factor model is already close to the true *EC* values obtained by simulations. Our results hold for portfolios with different levels of sector concentration, a different number of sectors as well as under various sector weight and correlation assumptions. Furthermore, we explore the accuracy of our model when the two assumptions that the portfolio is infinitely granular within each sector and that all exposures in the same sector have the same PD are violated. We found that the model underestimates *EC* in cases of low granularity, whereas it overestimates *EC* in the presence of heterogeneity in individual PDs, in particular if creditworthiness increases with exposure size. Which of the two effects prevails depends on the specific input parameters. The results seem to suggest, however, that for typical credit portfolios, the effect of PD heterogeneity is likely to be stronger than the effect of granularity. This implies that the analytic approximations err on the conservative side.

To our knowledge there is only one recent empirical paper that considers the impact of sector concentration risk on economic capital. Burton et al (2005) simulated the distribution of portfolio credit losses for a number of real US syndicated loan portfolios. They found that, although name concentration can meaningfully increase *EC* for smaller portfolios (which are defined as portfolios with exposures of less than US$10 billion), sector concentration risk is the main contributor to *EC* for portfolios of all sizes.

Two other models that measure concentration risk in a tractable model are presented by Cespedes et al (2005) and Düllmann (2006). Cespedes et al (2005) developed an adjustment to the single risk factor model in the form of a scaling factor to the economic capital required by the ASRF model. This "diversification factor" is an approximately linear function of a Hirschmann-Herfindahl index, calculated from the aggregated sector exposures. This model, however, does not allow for different asset correlations across sectors. Contrary to the approach in our paper, it cannot distinguish between a portfolio which is highly concentrated towards a sector with a high correlation with other sectors, and another portfolio which is equally highly concentrated, but towards a sector which is only weakly correlated with other sectors. Düllmann (2006) extends Moody's Binomial Expansion Technique by introducing default infection into the hypothetical portfolio on which the real portfolio is mapped in order to retain a simple solution for VaR. Unlike the Pykhtin model, both models developed by Cespedes et al and Düllmann require the calibration of a parameter using Monte Carlo simulations.

The paper is organized as follows. In Section 2 we present the default-mode version of the well-established multi-factor CreditMetrics model which serves as a benchmark. Furthermore, we discuss the simplified version of the Pykhtin model.

The empirical part of our paper comprises Sections 3 and 4. The loan portfolios on which the empirical analyses are based are described in Section 3. In Section 4 we analyze the impact of sector concentration on *EC*. For this purpose we gradually increase sector concentration, starting from the benchmark portfolio, and analyze its impact on *EC*.

In the theoretical part, which comprises Sections 5 to 7, we evaluate the performance of the Pykhtin's (2004) analytic approximation for economic capital using *EC* estimates from Monte Carlo simulations as a benchmark. Section 5 focuses on highly granular portfolios which are homogeneous on a sector level and, in particular, on the sensitivity of the results to the number of selected factors. Section 6 deals with portfolios characterized by lower granularity and Section 7 introduces PD heterogeneity on an exposure level.

Section 8 summarizes and concludes.

2. Measuring Concentration Risk in a Multi-Factor Model

2.1. General framework

We assume that every loan in a portfolio can be assigned to a different borrower, so that the number of exposures or loans equals the number of borrowers. Each borrower i can uniquely be assigned to a single specific sector. In practice, (large) firms often comprise business lines from different industry sectors. However, we pose this assumption here for practical and presentational purposes. Let M denote the total number of borrowers or loans in the portfolio, M_s the number of borrowers or loans in sector s, and S the total number of sectors. Each exposure has the same relative portfolio weight of $w_{s,i} = 1/M$. Therefore, the weight of the aggregated sector exposure, w_s, always equals M_s/M.

The general framework is a multi-factor default-mode Merton-type model.[6] The dependence structure between borrower defaults is driven by sector-dependent systematic risk factors which are usually correlated. Each risk factor can be uniquely assigned to a different sector, so that the number of sectors and factors are the same. Credit risk occurs only as a default event which is consistent with traditional book-value accounting and forms the basis of traditional loan portfolio management. The unobservable, normalized asset return $X_{s,i}$ of borrower i in sector s triggers the default event if it crosses the default barrier $\gamma_{s,i}$. The unconditional default probability $p_{s,i}$ of borrower i in sector s is defined as

$$p_{s,i} = P\left(X_{s,i} \leq \gamma_{s,i}\right).$$

The latent variable $X_{s,i}$ follows a factor model and can be written as a linear function of an industry sector risk factor Y_s and an idiosyncratic risk factor $\varepsilon_{s,i}$:

$$\text{(1a)} \quad X_{s,i} = r_s Y_s + \sqrt{1 - r_s^2}\,\varepsilon_{s,i}.$$

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6 See also Gupton et al (1997), Gordy (2000), and Bluhm et al (2003) for more detailed information on this type of models. The origin of these models can be found in the seminal work by Merton (1974).

The higher the value of the factor weight r_s, the more sensitive the asset returns of firm i in sector s are to the sector factor. The disturbance term $\varepsilon_{s,i}$ follows a standard normal distribution. The sector weight also determines the factor weight of the idiosyncratic risk factor in order to retain a standard normal distribution for $X_{s,i}$.

The correlations between the systematic sector risk factors Y_s are denoted by $\rho_{s,t}$ and often referred to as factor correlations. The sector factors can be expressed as a linear combination of independent, standard normally distributed factors $Z_1,\ldots,Z_S$.

$$\text{(1b)} \quad Y_s = \sum_{j=1}^{S} \alpha_{s,j} Z_j \text{ with } \sum_{j=1}^{S} \alpha_{s,j}^2 = 1 \text{ for } 1 \le s \le S.$$

The matrix $\left(\alpha_{s,t}\right)_{1 \le s,t \le S}$ is obtained from a Cholesky decomposition of the factor correlation matrix. The asset correlation for each pair of borrowers i and j in sectors s and t can be shown to be given by

$$\text{(2)} \qquad cor(X_{s,i}, X_{t,j}) = r_s \ r_t \ \rho_{s,t} = r_s \ r_t \sum_{n=1}^{S} \alpha_{s,n} \alpha_{t,n} .$$

Dependencies between borrowers arise only from their affiliation with the industry sector and from the correlations between the systematic sectors factors. The intra-sector asset correlation for each pair of borrowers is simply the sector weight r_s^2 squared.

If a firm defaults, the amount of loss is determined by the stochastic loss severity $\psi_{s,i}$. We assume that the loss severity is known at default and that before this event it is subject only to fully diversified idiosyncratic risk.[7] Credit losses of the whole portfolio are then given by

$$\text{(3)} \qquad L = \sum_{s=1}^{S} \sum_{i=1}^{M_s} w_{s,i} \psi_{s,i} 1_{\left\{X_{s,i} \le N^{-1}(p_{s,i})\right\}} .$$

In summary, the model needs the following input parameters:

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[7] The models analyzed in this paper can also be extended to incorporate idiosyncratic risk in loss severities, if required.

- the relative exposure size $w_{s,i}$ of borrower i in sector s
- the default probability $p_{s,i}$ of borrower i in sector s
- the expected loss severity $\mu_{s,i} = E\left[\psi_{s,i}\right]$ of borrower i in sector s
- the factor correlation matrix and
- the sector weight r_s.

In the following we assume that the factor correlation matrix can be proxied by a correlation matrix of equity index returns. In theory, asset correlations could be directly estimated from the time series of asset values. However, asset values are usually not observable. Since equity can be viewed as a call option on a firm's assets, it is frequently argued that correlation estimates from equity returns provide a viable approximation of factor correlations.[8]

2.2. The CreditMetrics default-mode model

To obtain the loss distribution, CreditMetrics applies Monte Carlo simulations by generating asset returns and counting the default events. In each simulation run the portfolio loss is determined from equation (3). For each exposure, the asset returns are determined from equations (1a/b) and compared with the default threshold. If the value of the asset returns falls below the threshold $\gamma_{s,i} = N^{-1}(p_{s,i})$, the borrower is in default. The portfolio loss of a simulation run is calculated by adding up the incurred losses from defaulted borrowers. The number of simulation runs in our analyses is typically 500,000. Portfolio losses obtained in each simulation run are sorted to form the distribution of portfolio losses from which EC can be calculated as the difference between the q-quantile of this loss distribution and the expected loss. Since it is obtained by simulation, we refer to it in the following as EC_{sim}.

[8] Previous analyses (See, for example, Zeng and Zhang (2001)) have shown that equity correlations may not be the best proxies for asset correlations, given that equity correlations may be subject to noise which may not necessarily be related to firms' fundamentals. However, the main advantage of using equity data is that there is an abundance of data. Therefore, it is no surprise to see that it has become market practice to use equity correlation as a proxy for asset correlation.

2.3. Analytic EC approximation

In this section, we describe an analytical approximation to the VaR in the framework of a multi-factor model, which is a simplified version of the model developed by Pykhtin (2004). The main advantage of this model is its tractability, since it does not require Monte Carlo simulations. Furthermore, we have simplified the model in such a way that it only requires exposure size, PD, and expected loss severities aggregated on a sector level instead of on an exposure level. The factor correlation matrix and the factor loadings are still needed as in the CreditMetrics model.

On the basis of the work by Gouriéroux et al (2000) and Martin and Wilde (2002), we can approximate the “true” loss L of a portfolio by a perturbed loss variable $L_\varepsilon = \bar{L} + \varepsilon \cdot U$, where U can be interpreted as $L - \bar{L}$ and ε describes the scaling parameter in the perturbation. $\bar{L}$ denotes the loss in the asymptotic single risk factor (ASRF) model with infinitely granular sectors. It depends on the default probability $\hat{p}(\bar{Y})$ conditional on the systematic risk factor $\bar{Y}$

$$(4) \qquad \bar{L} = \sum_{s=1}^{S} w_s \mu_s \hat{p}_s\left(\bar{Y}\right) \text{ with } \hat{p}_s(\bar{Y}) = N\left(\frac{N^{-1}(p_s) - c_s\bar{Y}}{\sqrt{1-c_s^2}}\right).$$

The parameter c_s can be interpreted as the correlation between the systematic risk factor $\bar{Y}$ and the asset returns $X_{s,i}$. The *q*-quantile of the “true” loss distribution, $t_q(L)$ can be approximated by $t_q\left(L_\varepsilon\right)$ as the sum of the VaR in the ASRF model, $t_q(\bar{L})$ and a multi-factor adjustment. This multi-factor adjustment can be determined from a second-order Taylor series expansion of $t_q(L)$. The first-order effect vanishes because we require $\bar{L} = E\left[L \mid \bar{Y}\right]$. By keeping terms up to quadratic and neglecting higher-order terms, we can approximate $t_q(L)$ as follows:[9]

$$(5) \qquad t_q(L) \approx t_q(\bar{L}) + \frac{1}{2}\frac{d^2 t_q(L_\varepsilon)}{d\varepsilon^2}\bigg|_{\varepsilon=0}$$

[9] See Pykhtin (2004) for proofs.

The first summand in (5) denotes the VaR in the single risk factor model and can be calculated by replacing $\bar{Y}$ in (4) by $t_q\left(\bar{Y}\right)$. If $\bar{Y}$ follows a standard normal distribution, $t_q\left(\bar{Y}\right)$ is the q-quantile of this distribution. Note that this single risk factor model differs from the well-known ASRF model in that $\hat{\rho}_s\left(t_q\left(\bar{Y}\right)\right)$ depends on a sector-dependent asset correlation. To avoid confusion, we will call this model the "ASRF* model", reserving the term "ASRF model" for the model with uniform asset correlations.

The second summand in (5) denotes the multi-factor adjustment, Δt_q, which can be calculated according to Pykhtin (2004) by

$$(6) \qquad \Delta t_q = -\frac{1}{2l'(y)}\left[v'(y) - v(y)\left(\frac{l''(y)}{l'(y)} + y\right)\right]\Bigg|_{y=N^{-1}(1-q)}$$

where $l'(y)$ and $l''(y)$ denote, respectively, the first and second derivative of equation (4). $v(y)$ gives the conditional variance of $L-\bar{L}$ (conditional on $y=\bar{Y}$) which equals the conditional variance of L. Its first derivative is $v'(y)$. The details and the inputs of these equations are presented in Appendix B.

The remaining open issue is how to establish the link between L and $\bar{L}$. This is achieved by restricting $\bar{Y}$ to the space of linear mappings of the risk factors $Z_1,\ldots Z_S$:

$$\bar{Y} = \sum_{s=1}^{S} b_s Z_s \ .$$

The correlations between the industry risk factors Y_s and the systematic risk factor $\bar{Y}$ are denoted by $\bar{\rho}$. These are used to calculate the (also sector-dependent) correlations in the ASRF* model using the following mapping function, for $s \in \{1,\ldots,S\}$:

$$c_s = r_s \bar{\rho} \ \text{ where } \ \bar{\rho} = \sum_{j=1}^{S} \alpha_{s,j} b_j$$

There is no unique solution for determining the coefficients $b_1,\ldots,b_S$. In the following, we will use the approach in Pykhtin (2004), which is briefly summarized in Appendix C.

3. Portfolio Composition

3.1. Data set and definition of sectors

Our analyses are based on loan portfolios which reflect characteristics of real bank portfolios obtained from European credit register data. Our benchmark portfolio represents the overall sector concentration of the German banking system which was constructed by aggregating the exposure values of loan portfolios of 2224 German banks in September 2004. The portfolio includes branches of foreign banks located in Germany. Credit exposures to foreign borrowers, however, are excluded. We deem this to be a reasonable approximation of a well-diversified portfolio on the intuitive assumption that a portfolio cannot be more diversified than in the case in which it represents the average relative sector exposures of the national banking system. In principle, we could also have created a more diversified portfolio in the sense of having a lower VaR. However, such a portfolio would be specific to the credit risk model used and would not be obtainable for all banks.

All credit institutions in Germany are required by the German Banking Act (*Kreditwesengesetz*) to report quarterly exposure amounts of those borrowers whose indebtedness to them amounts to €1.5 million or more at any time during the three calendar months preceding the reporting date. In addition, banks report national codes that are compatible with the NACE classification scheme and indicate the economic activity of the borrower and his country of residence. Individual borrowers are summarized in *borrower units* which are linked, for example, by investments and constitute an entity sharing roughly the same risk. The aggregation of exposures on a business sector level was carried out on the basis of borrower units. If borrowers in that unit belong to different sectors, the dominating exposure amount determines the final sector allocation. Therefore, the credit register includes not only exposures above €1.5 million, but also smaller exposures to individual borrowers belonging to a borrower unit that exceeds this exposure limit. This characteristic substantially increases its coverage of the credit market.

The industry classification chosen by CreditMetrics is the Global Industry Classification Standard (GICS), which was jointly launched by Standard & Poor's and Morgan Stanley Capital International (MSCI) in 1999. The classification scheme was developed to establish a global standard for categorizing firms into sectors and industries according to their principal business activities. It comprises 10 broad sectors which are divided into 24 industry groups.[10] GICS further divides these groups into industries and sub-industries. However, the latter detailed schemes are not used by vendor models. In the following, we use the broad sector classification scheme. Because some of the industry groups that form the broad "Industrial" sector are very heterogeneous, we decided to split this sector into three industry groups: Capital Goods (including Construction), Commercial Services and Supplies, and Transportation.[11]

Credit register datasets, however, use the NACE industry classification system, which is quite different from the GICS system. In order to use the information from the credit register, we mapped[12] the NACE codes onto the GICS codes. Similar mapping is used by other vendor models, such as S&P's Portfolio Risk Tracker developed by S&P. We have excluded exposures to the financial sector (sector G) which comprises exposures to Banks (G1), Diversified Financials (G2), Insurance Companies (G3) and Real Estate (G4). We have excluded exposures to the financial sector because of the specificities of this sector. Exposures to the real estate sector are heavily biased as it comprises a large number of exposures to borrowers that are related to the public sector. Since we could not differentiate between private and public enterprises in the real estate sector, we have excluded this sector from the following analyses. We have also disregarded exposures to households since there is no representative stock index for them. This is a typical limitation of models relying on equity data for the estimation of asset correlations. In sum, we distinguish between 11

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[10] See Table 12 in Appendix A, which shows the broad sectors and the more detailed industry groups.

[11] Unreported simulations have shown that results are not affected by using the more detailed classification scheme.

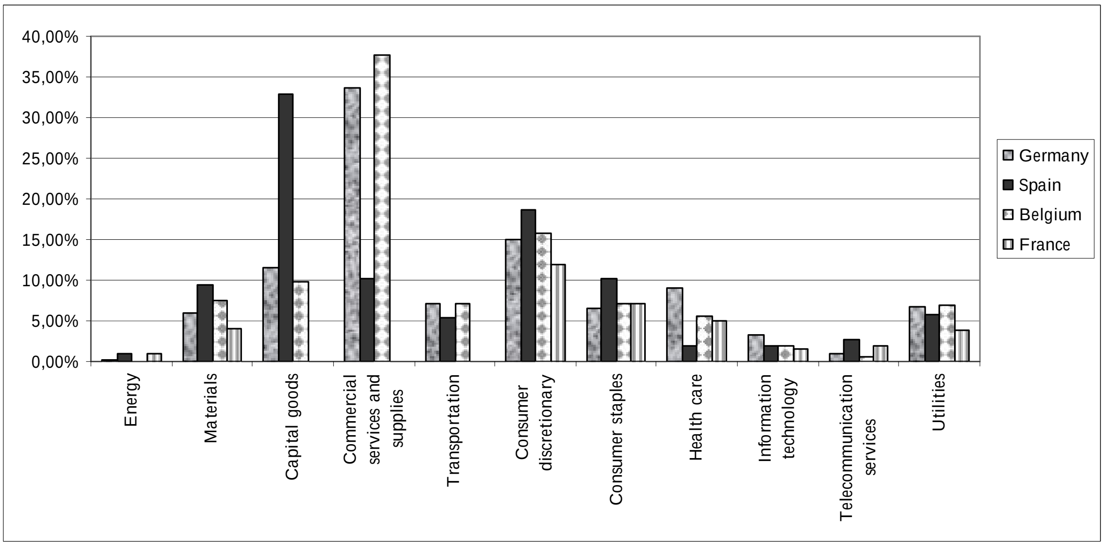
[12] See Table 13 in Appendix A for the mapping.

sectors, which can be considered as broadly representing the Basel II asset classes Corporate and SMEs.

3.2. Comparison with French, Belgian and Spanish banking systems

A rough comparison of the relative share of the sector decomposition between the aggregated German, French, Belgian and Spanish banking systems shows that the numbers are similar.[13] The only noticeable difference is the greater share of the Capital Goods sector (33%) in Spain compared to Germany and Belgium and the smaller share of the Commercial Services and Supplies sector in Spain compared to Germany and Belgium. In general, however, the average sector concentrations are very similar across the four countries, which suggests that our results are to a large extent transferable.

Figure 1: Comparison of average sector concentration for Germany, Spain, Belgium, and France (*)



(*) A breakdown of Industrial sector C into the three categories Capital Goods, Commercial Services and Supplies, and Transportation is not available for France. The sector shares of the aggregated sector C, however, are quite similar for all four countries.

13 The exact figures are provided by Table 14 in Appendix A.

3.3. Description of the benchmark portfolio

The sectoral distribution of exposures in the benchmark portfolio, which is shown in Table 1, assumes that the total portfolio has a volume of €6 million. As mentioned above, this portfolio represents the sectoral distribution of aggregate exposures in the German banking system. The degree of concentration in our reference portfolio is purely national and driven by the firms' sector composition because we do not consider the impact of regional or country factors in our analysis. It is not uncommon for banks to use a more detailed sector classification scheme. We consider it more conservative to use a relatively broad sector classification scheme rather than a very detailed one. In a broad sector classification scheme, a larger proportion of exposures is attached to one sector. Therefore, correlations between exposures of the same sector, which are typically greater than the correlations between exposures of a different sector, will play a larger role.

In order to focus on the impact of sector concentration we assume an otherwise homogeneous portfolio by requiring that all other characteristics of the portfolio are uniform across sectors. We assume a total portfolio volume of €6 million that consists of 6,000 exposures of equal size and a uniform probability of default (PD) of 2%. Every exposure is to a different borrower, thus circumventing the need to consider multiple exposure defaults. We set a uniform LGD of 45%, which is the corresponding supervisory value for a senior unsecured loan in the Foundation IRB approach of the Basel II framework.[14] In the CreditMetrics approach, industry weights can be assigned to each borrower according to its participation. Here, we assume that every firm is exposed to only one single sector as its main activity. Furthermore, we assume banks do not reduce exposure to certain sectors by purchasing credit protection.

[14] See BCBS (2004b).

Table 1: Composition of the benchmark portfolio (using the GICS sector classification scheme)

	Total exposure	Number of exposures	% exposure
A: Energy	11,000	11	0.18%
B: Materials	361,000	361	6.01%
C1: Capital Goods	692,000	692	11.53%
C2: Commercial Services and Supplies	2,020,000	2,020	33.69%
C3: Transportation	429,000	429	7.14%
D: Consumer Discretionary	898,000	898	14.97%
E: Consumer Staples	389,000	389	6.48%
F: Health Care	545,000	545	9.09%
H: Information Technology	192,000	192	3.20%
I: Telecommunication Services	63,000	63	1.04%
J: Utilities	400,000	400	6.67%
Total	6,000,000	6,000	

3.4. Sequence of portfolios with increasing sector concentration

In order to measure the impact on *EC* of more concentrated portfolios than the benchmark portfolio, we construct a sequence of six portfolios, each with increased sector concentration relative to the previous one. To this end, we gradually increase sector concentration in our benchmark portfolio by using the following algorithm. In each step we remove x exposures from all sectors and add them to a previously selected sector. This procedure is repeated until a single-sector portfolio which is the portfolio with the highest possible concentration is obtained. The sector which receives x exposures at every step and also the amount x that is transferred to this sector are determined in such a way that some of the generated portfolios reflect a degree of sector concentration that is actually observable in real banks.[15]

Table 2 shows a sequence of seven portfolios in the order of increasing sector concentration. The increase in sector concentration is also reflected in the Herfindahl-Hirschmann Index (HHI),[16] given in the last row which is calculated at sector level. Portfolio 1 has been constructed from the benchmark portfolio by re-allocating one third of each sector exposure to the sector Capital Goods. The even more concentrated portfolios 2, 3, 4 and 5 have been created by repeated application of this rule. Portfolios 2 and 5 are similar to portfolios of

15 Due to confidentiality requirements, we are unable to reveal more detailed information.

16 See Hirschmann (1964).

existing banks[17] insofar as the sector with the largest exposure size has a similar share of the total portfolio. Furthermore, the HHI is similar to what is observed in real-world portfolios. Finally, we created portfolio 6 with the highest degree of concentration as a one-sector portfolio by shifting all exposures to the Capital Goods sector.

Table 2: Sequence of portfolios with increasing sector concentration

	Benchmark portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
A: Energy	0%	0%	0%	0%	0%	0%	0%
B: Materials	6%	4%	3%	2%	2%	1%	0%
C1: Capital Goods	12%	41%	56%	71%	78%	82%	100%
C2: Commercial Services & Supplies	34%	22%	17%	11%	8%	7%	0%
C3: Transportation	7%	5%	4%	2%	2%	1%	0%
D: Consumer Discretionary	15%	10%	7%	5%	4%	3%	0%
E: Consumer Staples	6%	4%	3%	2%	2%	1%	0%
F: Health Care	9%	6%	5%	3%	2%	2%	0%
H: Information Technology	3%	2%	2%	1%	1%	1%	0%
I: Telecommunication Services	1%	1%	1%	0%	0%	0%	0%
J: Utilities	7%	4%	3%	2%	2%	1%	0%
HHI	17.6	24.1	35.2	51.5	61.7	68.4	1

3.5. Intra and inter-sectoral correlations

The sector factor correlations are estimated from historical equity index correlations. Table 2 shows the equity correlation matrix of the relevant MSCI EMU industry indices.[18] The sector factor correlations are based on weekly return data covering the period from November 2003 to November 2004. Sectors that are highly correlated with other sectors (i.e. sectors that have an average inter-sector equity correlation of greater than 65%) are Materials (B), Capital Goods (C1), Transportation (C3) and Consumer Discretionary (D). Sectors that are moderately correlated with other sectors, i.e. sectors that have an average inter-sector equity correlation of between 45% and 65%, are Commercial Services and Supplies (C2), Consumer Staples (E) and Telecommunication (I). Sectors that are the least correlated with other sectors, i.e. sectors that have an average inter-sector equity correlation of less than

[17] Confidentiality requires those banks with a high sector concentration remain anonymous.

[18] The correlation matrix based on MSCI US data is similar.

45%, are Energy (A) and Health Care (F). The relative order of these sectors is broadly in line with results reported in other empirical papers.[19] The heterogeneity between Capital Goods, Commercial Services and Supplies, and Transportation are confirmed by noticeable differences in correlations. The intra-sector correlations and/or inter-sector correlations between exposures are obtained by multiplying these sector correlations of Table 3 with the sector weights.

Table 3: Correlation matrix based on MSCI EMU industry indices (based on weekly log return data covering the Nov 2003 - Nov 2004 period; in percent)

	A	B	C1	C2	C3	D	E	F	H	I	J
A: Energy	100	50	42	34	45	46	57	34	10	31	69
B: Materials		100	87	61	75	84	62	30	56	73	66
C1: Capital Goods			100	67	83	92	65	32	69	82	66
C2: Commercial Services & Supplies				100	58	68	40	8	50	60	37
C3: Transportation					100	83	68	27	58	77	67
D: Consumer discretionary						100	76	21	69	81	66
E: Consumer staples							100	33	46	56	66
F: Health Care								100	15	24	46
H: Information Technology									100	75	42
I: Telecommu-nication Services										100	62
J: Utilities											100

More difficult than the estimation of sector correlations is the determination of the sector weights, which depend also on the intra-sector asset correlations. We do not use the formula provided in CreditMetrics to determine the sector weights as recent research has suggested that this formula does not fit the German data very well.[20] Instead, we assume a unique sector weight for all exposures and calibrate the value of the factor loading to match the corresponding IRB regulatory capital charge. More precisely, we determine a factor loading $r_s=0.50$ for all sectors $s \in \{1,...,S\}$ such that the economic capital EC_{sim} equals the IRB capital charge for corporate exposures, assuming

[19] See, for example, De Servigny and Renault (2001), FitchRatings (2004) and Moody's (2004). It is difficult to compare the absolute inter-sector correlation values as different papers report different types of correlations. De Servigny and Renault (2001) report inter-sector default correlation values, FitchRatings (2004) reports inter-sector equity correlations while Moody's (2004) provides correlation estimates inferred from co-movements in ratings and asset correlation estimates. Furthermore, the different papers distinguish between a different number of sectors.

[20] See Hahnenstein (2004) for a detailed analysis.

a default probability of 2%, a loss given default (LGD) of 45% and a maturity of one year.

Setting the sector factor weight to 0.5 is slightly more conservative than empirical results for German companies suggest. The average of all the correlation entries in the factor correlation matrix is 0.59, which implies by evoking equation (2) an average asset correlation of 0.14 between exposures. Empirical evidence[21] has shown that German SMEs typically have an average asset correlation of 0.09, which implies that $r_s = 0.5$. Large firms, however, are typically more exposed to systematic risk than SMEs and therefore usually have higher asset correlation values.[22]

Equation (2) implies that intra-sector asset correlations are thus fixed at 25%. Inter-sector asset correlations can be calculated by multiplying the factor weights of both sectors by the inter-sector equity correlation. The lowest equity correlation between the Energy sector index and the Information Technology sector index of 10% translates into inter-sector asset correlations of 2.5%. The highest equity index correlation occurs between the Commercial Services and Supplies and the Consumer Discretionary sector index. At 92%, it translates into an inter-sector asset correlation of 23%.

4. Impact of sector concentration on economic capital

In this section we analyze the impact of increasing sector concentration on economic capital, which is defined as the difference between the 99.9% percentile of the loss distribution and the expected loss. The results are given in Table 4. We observe for the corporate portfolios that economic capital increases from the benchmark portfolio to portfolio 2 by 20%. Economic capital for the concentrated portfolio 5 increases by a substantial 37% relative to the benchmark portfolio. These results demonstrate the importance of taking sector concentration into account when calculating *EC*.

Typically, the corporate portfolio comprises only a fraction of the total loan portfolio (which also contains loans to sovereigns, other banks and private

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21 See Hahnenstein (2004).

22 See, for example, Lopez (2004) for empirical evidence of this relation for the US.

retail clients). Although the increase in sector concentration may have a significant impact on the economic capital for the corporate credit portfolio, it may have a much smaller impact in terms of a bank's total credit portfolio. For a meaningful comparison, we assume that the corporate credit portfolio comprises 30% of the total credit portfolio and that the banks need to hold capital amounting to 8% of their total portfolio. By assuming that there are no diversification benefits between corporate exposures and the bank's other assets, the *EC* of the total portfolio can be determined as the sum of the *EC* for the corporate exposures and the *EC* for the remaining exposures.

Table 4: Impact of sector concentration on economic capital (EC_{sim}) for the sequence of corporate portfolios and for the sequence of total portfolios of a bank (in percent of total exposure)

	Benchmark portfolio	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
Corporate portfolio	7.8	8.8	9.5	10.1	10.3	10.7	11.7
Total portfolio	8.0	8.2	8.5	8.7	8.8	8.9	9.2

The results for the total portfolios of the bank are also shown in Table 4. As expected, the impact of an increase in sector concentration is much less severe when looking at the *EC* for the total portfolio. Economic capital for portfolio 5, for example, increases by about 16% relative to the benchmark portfolio instead of 37% if only the corporate portfolio is taken into account.

In order to verify how robust our results are to the input parameters, we carried out the following four robustness checks (RC1 - RC4):

- a lower uniform PD of 0.5% instead of 2% for all sectors (RC1),
- heterogeneous PDs estimated from historical default rates of the individual sectors (RC2) and given in Table 5,
- a different factor correlation matrix (See Table 15, Appendix A) representing the correlation matrix with the highest average annual correlation over the period between 1997 and 2005 (RC3) and

- a uniform intra-sector asset correlation of 15% and a uniform inter-sector asset correlation of 6% (RC4), which are values used by Moody's for the risk analysis of synthetic CDOs.[23]

Table 5: Average historical default rates (1990-2004; before and after scaling to an exposure-weighted expected average default rate of 2% for the benchmark portfolio; in percent)

Sector	Unscaled default rate	Scaled default rate
A: Energy	1.5	1.0
B: Materials	2.8	1.9
C1: Capital Goods	2.9	2.0
C2: Commercial Services and Supplies	3.7	2.5
C3: Transportation	2.9	2.0
D: Consumer Discretionary	3.2	2.2
E: Consumer Staples	3.5	2.4
F: Health Care	1.6	1.1
H: Information Technology	2.4	1.6
I: Telecommunication Services	3.6	2.4
J: Utilities	0.6	0.4

Source: own calculation, based on S&P (2004)

The historical default rates in Table 5 are, on average, higher than the value of 2% which is used for the PDs in the case of homogeneous PDs for all sectors. In order to isolate the effect of PD heterogeneity between sectors, we scale the historical default rate, p_s^{hist}, for every sector s as follows,

$$(7) \qquad p_s^{scaled} = p_s^{hist} \frac{0.02}{\sum_{s=1}^{S} w_s \cdot p_s^{hist}}.$$

In this way we ensure that the weighted average PD of the benchmark portfolio stays at 2% even in the case of PD heterogeneity across sectors.

The results of the four robustness checks are summarized in Table 6. Although the absolute level of EC varies between these robustness checks, the relative increase in EC compared with the benchmark portfolio is similar to previous results in this section. For Moody's correlation assumptions in RC4, the increase in EC is stronger than for the other robustness checks. This can be explained by the larger difference between intra-sector and inter-sector correlations, which is justified by the higher number of sectors they use, and

[23] See Fu et al (2004).

which leads to a stronger *EC* increase when the portfolio becomes more and more concentrated in a single sector. We conclude that the observed substantial relative increase in *EC* due to the introduction of sector concentration is robust against realistic variation of the input parameters. Furthermore, this increase in *EC* may be even greater, depending on the underlying dependence structure.

Table 6: EC for the benchmark portfolio and its relative increase for the more concentrated portfolios 1 - 6 (in percent of total exposure)

Portfolio	Using "Real-rule"	RC1: PD=0.5%	RC2: Heterogeneous (scaled) PD	RC3: Higher correlation	RC4: Moody's
EC					
Benchmark portfolio	7.8	3.3	8.0	8.7	4.0
Proportional change of EC in %					
Portfolio 1	+13	+12	+11	+6	+6
Portfolio 2	+20	+21	+18	+13	+18
Portfolio 3	+30	+29	+26	+22	+39
Portfolio 4	+35	+37	+31	+24	+46
Portfolio 5	+36	+42	+34	+24	+51
Portfolio 6	+49	+52	+45	+33	+77

5. Evaluation of the EC Approximations for Sector-dependent PDs and High Granularity

The purpose of this section is to analyze the performance of the *EC* approximations, given homogeneity within each sector and assuming a highly granular exposure distribution in each sector. Since these are two model assumptions, the results can be understood as an upper bound in terms of approximation quality. The analysis is performed by varying one-by-one the sector distributions, the factor correlations, the factor weights, the number of factors and the sector PD. Cases of less granular portfolios and heterogeneous PDs on an exposure level are studied in Sections 6 and 7.

We again assume a confidence level q of 99.9% and employ the following three risk measures (where $EL = \sum_{s=1}^{S} \sum_{i=1}^{M_s} w_{s,i} \psi_{s,i} p_{s,i}$):

- economic capital in the ASRF* model, which is defined as $EC^{*} = t_{99.9\%}\left(\bar{L}\right) - EL$

- economic capital based on the multi-factor adjustment, $EC_{MFA} = t_{99.9\%}\left(\bar{L}\right) + \Delta t_{99.9\%} - EL$

- economic capital based on Monte Carlo (MC) simulations, EC_{sim}

Firstly, we present results for the benchmark portfolio and for the more concentrated portfolios 1 - 6 in Table 7. The model parameters are the same as in Section 4.

Table 7: Comparison of EC^{*}, EC_{MFA} and EC_{sim} for different exposure distributions across sectors with increasing sector concentration given a default probability of 2% (in percent of total exposure)

Portfolio	EC^{*}	EC_{MFA}	EC_{sim}	Relative error of EC_{MFA}
Benchmark portfolio	7.8	7.9	7.8	1.3%
Portfolio 1	8.7	8.8	8.8	0.0%
Portfolio 2	9.4	9.4	9.5	-1.1%
Portfolio 3	10.1	10.1	10.1	0.0%
Portfolio 4	10.5	10.5	10.3	1.9%
Portfolio 5	10.7	10.7	10.7	0.0%
Portfolio 6	11.6	11.6	11.7	-0.9%

The EC figures for the benchmark portfolio in Table 7 show that EC^{*} and EC_{MFA} provide extremely accurate proxies for EC_{sim}. This result suggests that in the given examples the calculation of EC^{*} may, in practice, be sufficiently accurate for certain risk-management purposes. The four EC estimates for the more highly concentrated portfolios 1 - 6 indicate that economic capital increases as expected, but that our results for the approximation performance of EC^{*} and EC_{MFA} still hold. According to Table 7, relative errors of EC_{MFA} are in a relatively small range between 0.0% and 1.3%.

Secondly, we check whether our results differ when we vary the underlying correlation structure. To this end we calculate in Table 8 the three risk measures for different factor correlation matrices. More specifically, we assume homogeneous factor correlation matrices in which the entries (outside the main diagonal) vary between 0 and 1 in increments of 0.2. The last case, in which all factor correlations are equal to one, corresponds to the case of a single-factor model.

Table 8: Comparison of EC^*, EC_{MFA} and EC_{sim} for different factor correlations ρ, given a default probability of 2% (in percent of total exposure)

Factor correlation ρ	EC^*	EC_{MFA}	EC_{sim}	Relative error of EC_{MFA}
0.0	3.3	3.9	4.0	-2.5%
0.2	4.5	4.9	5.0	-2.0%
0.4	6.1	6.3	6.3	0.0%
0.6	7.9	7.8	8.0	-2.5%
0.8	9.7	9.7	9.9	-2.0%
1.0	11.6	11.6	11.9	-2.5%

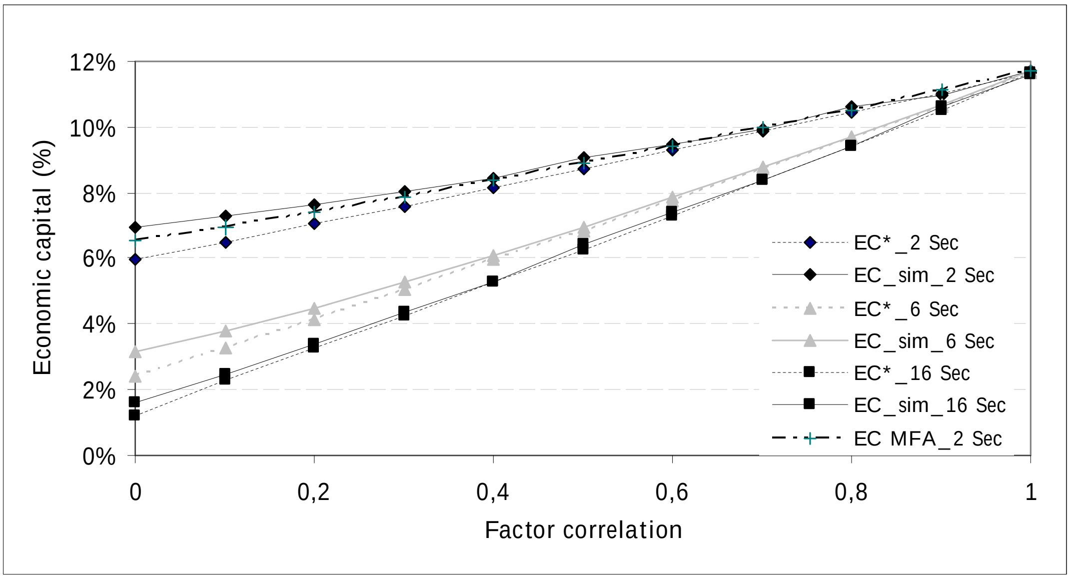
Table 8 shows EC_{sim} and its proxies EC^* and EC_{MFA} for increasing factor correlations. As expected, economic capital increases with increasing factor correlations, since a higher factor correlation reduces the diversification potential by shifting probability mass to the tail of the loss distribution. The highest relative error of EC_{MFA} of all factor correlations considered is 2.5% which still reveals a good approximation performance. With increasing factor correlations the multi-factor model approaches the structure of a one-factor model for which EC^* and EC_{MFA} coincide. In all cases EC^* is relatively close to EC_{MFA}. Therefore, our earlier results concerning the good approximation performance of EC^* and EC_{MFA} also hold under different factor correlation assumptions.

Thirdly, we relax the assumption that the factor weight r is fixed by increasing the factor weight r from 0.2 to 0.8. There is a strong increase in EC with the factor weight but this does not affect the approximation quality, neither of EC^* nor of EC_{MFA}.

Fourthly, we explore how the results depend on the number of factors. For this purpose we vary the number of factors from 2 to 16. Figure 2 shows how EC^*, EC_{MFA} and EC_{sim} depend on the number of sectors and the factor correlation.

EC_{MFA} is only plotted for 2 sectors because its values are indistinguishable from EC_{sim} for 6 and for 16 sectors.

Figure 2: Economic capital (EC^*, EC_{MFA} and EC_{sim}) for different factor correlation values for 2, 6 and 16 sectors (in percent of total exposure)



For a given number of sectors, EC increases in Figure 2 with factor correlation as expected. If the factor correlation approaches one, then the EC values coincide, irrespective of the number of sectors. The reason is that in the limiting case of a factor correlation equal to one, the model collapses to a single-factor model.

For a factor correlation of 0.6, which is also the average of the entries in the correlation matrix in Table 3, and also for higher factor correlations, the relative approximation error is below 1% for EC_{MFA} and below 2% for EC^*. Therefore, the previous results showing a good approximation performance of EC^* and an even better one for EC_{MFA} are found to be robust with respect to the number of sectors, at least for realistic factor correlations.

Figure 2 also shows that EC^* and EC_{sim} generally decrease when the number of sectors increases for given asset correlation values. This result can be explained by risk reduction through diversification across sectors.

Fifthly, we tested whether our results for the approximation performance of EC^* and EC_{MFA} are sensitive to PD heterogeneity on a *sector level.* For this purpose we employ the scaled default rates for sectors from Table 5. The results are given in Table 9.

Table 9: Comparison of EC^*, EC_{MFA} and EC_{sim}, based on sector-dependent default probabilities, estimated from historical default rates (in percent of total exposure)

Portfolio	EC^*	EC_{MFA}	EC_{sim}	Relative error of EC_{MFA}
Benchmark portfolio	8.0	8.0	8.0	0.0%
Portfolio 1	8.8	8.9	8.8	1.1%
Portfolio 2	9.4	9.4	9.5	-1.1%
Portfolio 3	10.1	10.1	10.1	0.0%
Portfolio 4	10.5	10.5	10.4	1.0%
Portfolio 5	10.7	10.7	10.7	0.0%
Portfolio 6	11.6	11.6	11.7	-0.9%

For all risk measures the results in Table 9 are relatively close to those in Table 7. The more concentrated the exposures are in one sector, however, the smaller the difference to Table 7 is. This is explained by the fact that the sector PDs are calibrated to an average value of 2% which is also the PD used for Table 7. The approximation quality of EC^* and EC_{MFA} is similar to Table 7. We conclude that, in qualitative terms, the results obtained for a uniform PD also hold for heterogeneous sector-dependent PDs.

6. Evaluation of the EC Approximations for Sector-dependent PDs and Low Granularity

Simulation results in the previous section, which reveal a reasonably good approximation quality for EC^* and EC_{MFA}, were obtained conditional to a uniform PD in every sector and highly granular portfolios because each individual exposure has a relatively small share of 0.017% (=1/6000) of the total portfolio volume. However, portfolios of small banks, in particular, are less granular. In the following we explore the impact of lower granularity. From the set of seven portfolios, only the benchmark portfolio and portfolio 6 are considered since they have the lowest and the highest sector concentration. The impact of granularity is considered in two cases.

In the first case, characterized by a portfolio of representative granularity, the distribution of exposure size was selected from a sample of typical small, regional German banks to reflect an average granularity in terms of the HHI. The purpose is to measure the impact of granularity for an exposure distribution that is representative for real banks. However, since the exposure distribution is based on central credit register data, only larger exposures are captured[24] in the underlying data set with the consequence that this exposure distribution is less granular than what we can expect for real bank portfolios. The HHI of the portfolio is 0.0067 and the descriptive statistics on exposure size are shown in Table 16 in Appendix D. The assignment of exposures to sectors was achieved by randomly drawing exposures from the data set such that exposure size follows the same distribution in every sector.

In the second case, characterized by *low granularity*, we consider the highest individual exposure shares that are admissible under the EU large exposure rules.[25] In this way we obtain an upper limit for the potential impact of granularity. According to the EU rules, an exposure is considered "large" if its amount requires 10% or more of regulatory capital. Banks are generally not allowed to have an exposure that requires at least 25% of regulatory capital. Furthermore, the sum of all large exposures must not require more than 8 times the regulatory capital.[26]

We assume that a bank's regulatory capital is 8% of its total loan volume. For a total portfolio value of 6,000 currency units, banks are required to hold 480 currency units in capital. Each large exposure requires a minimum amount of capital of 48 currency units and a maximum amount of 120 currency units. The total sum of all large exposures must not exceed 3,840 currency units. With these restrictions, the least granular admissible exposure distribution of our portfolio consists of

- 3840/120=32 loans of 120 currency units

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[24] See section 3.1 for more information on the characteristics of exposures included in the German central credit register.

[25] See Directive 93/6/EEC of 15 March 1993 on the capital adequacy of investment firms and credit institutions.

[26] The last two restrictions may be breached with permission of the German Federal Financial Supervisory Authority (BaFin), in which case the excess must be fully backed by capital.

- 2160/47= 45 loan exposures of 47 currency units (just below the large exposure limit of 48) and
- a remaining single exposure of 45 currency units

While keeping the average sector concentration of the portfolio constant, we increase the granularity of the portfolio to reflect the exposure size distribution of this least granular portfolio. More details of this portfolio can be found in Table 17, Appendix D. Simulated economic capital, EC_{sim}, and the analytic proxies EC^* and EC_{MFA} are given in Table 10.

Table 10: Comparison of EC^*, EC_{MFA} and EC_{sim} for portfolios with representative and low granularity (in percent of total exposure)

Portfolio	Granularity	EC^*	EC_{MFA}	EC_{sim}	Relative error of EC_{MFA}
Benchmark portfolio	representative	8.1	8.2	8.8	-7%
	low	8.0	8.0	9.3	-14%
Single sector portfolio	low	11.6	11.6	12.7	-8%

In Table 10 EC^* and EC_{MFA} for the representative granular portfolio are slightly higher than for the low granular portfolio. This difference is caused by minor differences in the exposure distribution across sectors which arise when the representative discrete exposure distribution is mapped to the sector allocation of the benchmark portfolio.

The EC_{sim} value of 9.3% for the low granular benchmark portfolio is 1.3 percentage points (or 14% in relative terms) higher than for the highly granular benchmark portfolio in Table 9. This difference appears to be substantial, but we have to consider that the granularity of the portfolio in Table 10 is very low since it reflects the lowest granularity permissible under European bank regulation. The EC_{sim} measure for the single sector portfolio 6 in Table 10 is higher than for the benchmark portfolio, which is consistent with earlier reported results.

The EC_{sim} value of 8.8% for the benchmark portfolio with typical granularity is relatively close to the value of 9.3% for the portfolio with low granularity, at least if compared with EC_{sim} of 8.0% for the infinitely granular benchmark portfolio in Table 9. One reason is that some exposures in the portfolio with typical granularity technically violate the large exposure rules.[27] Therefore, as mentioned before, the portfolio of "representative" granularity should still be regarded as conservative in terms of granularity.

For the purpose of this analysis, the approximation errors of the EC proxies, EC^* and EC_{MFA}, are more important than the level of EC. Both EC proxies are based on the assumption of infinite granularity in each sector, while the EC_{sim} calculations take into account PD heterogeneity across sectors and granularity. We find that EC^* and EC_{MFA} can substantially underestimate EC by up to 14%, in particular for portfolios with low granularity .

7. Evaluation of EC Approximations for Heterogeneous Sectors

So far we have only considered sector-dependent PDs, which means PD variation on a sector level, but not on the exposure level. In the following we explore the impact of heterogeneous PDs inside a sector together with the impact of granularity. For the benchmark portfolio of representative granularity analyzed in the previous section, exposure-dependent PDs were computed from a logit model based on firms' accounting data. In order to apply the logit model, borrower information from the central credit register on exposure size had to be matched with a balance sheet database, also maintained by the Deutsche Bundesbank.[28] Using empirical data on exposure size and PD automatically captures a potential dependence between these two characteristics. In order to ensure comparability with previous results, we apply the same scaling procedure as in Section 6 to achieve the same average sector PD. Information on this PD distribution is given in Table 18, Appendix D.

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27 This can be explained either by special BaFin approval or, most likely, by data limitations given that our credit register data do not contain loans below €1.5 million. The latter implies that their sum is lower than the total portfolio exposure of the data, providing real bank and, therefore, our relative exposure weights are biased upwards. In other words, it is well possible that the large exposure limit is breached because the total exposure as reference is downward biased, although the limit is still met by the data-providing bank.

28 More details on the database and the logit model that was used to determine the PDs can be found in Krüger et al. (2005).

The portfolio with the lowest granularity admissible under the EU large exposure rules is an artificially generated portfolio, so that we have no PD information for single exposures. Therefore, we randomly assign PDs from an empirical aggregate PD distribution based on the same balance sheet database, but this time aggregated over a sample of banks. The empirical PD distribution is given in Table 20 and information on the PD distribution of the low granular portfolio is provided in Table 19, Appendix D.[29]

The results for PD heterogeneity in every sector are given in Table 11. The reduction of EC_{sim} compared to Table 10, which occurs for both portfolios, is due to the PD heterogeneity on the exposure level. This impact of PD heterogeneity has also been noted by Hanson et al (2005) and can be explained by the concavity of the dependence of EC on PD.

Table 11: Comparison of EC^*, EC_{MFA} and EC_{sim} for portfolios with heterogeneous sectors (in percent of total exposure)

Portfolio	Granularity	EC^*	EC_{MFA}	EC_{sim}	Relative error of EC_{MFA}
Benchmark portfolio	representative	8.1	8.2	8.1	1%
	low	8.0	8.0	8.5	-6%
Single sector portfolio	low	11.6	11.6	10.8	+8%

Since EC^* and EC_{MFA} do not account for PD heterogeneity on the exposure level, these values stay unchanged from Table 10 while EC_{sim} decreases. As a consequence the underestimation by using EC^* and EC_{MFA} instead of EC_{sim} is reduced relative to Table 10. This is confirmed by the approximation error in the last column of Table 11, which is lower when using heterogeneous PDs compared to the case of sector-dependent PDs in Table 10. For the portfolio of representative granularity, EC^* and EC_{MFA} approximate EC extremely well as the relative error of EC_{MFA} is only 1%.

[29] Since a negative correlation between exposure size and PD emerged as a stylized fact in recent empirical literature (See, for example, Dietsch and Petey (2002) or Lopez (2004)), we also considered the case that the PDs are perfectly ordered in terms of decreasing exposure size. We found that our results are robust in this case.

In the single-sector portfolio, the approximation errors of the *EC* proxies are positive, implying that the effect of PD heterogeneity is stronger than the granularity effect, measured relative to the highly granular portfolio with homogeneous sector PDs. As a consequence, the ASRF* model actually overestimates *EC*.

In summary, the approximation errors for all portfolios considered vary between -6% and +8%. For the benchmark portfolio they are always lower than the corresponding values in Table 10 and for the single-sector portfolio the estimates are more conservative. The results of Table 10 and Table 11 taken together demonstrate that the effect of PD heterogeneity counterbalances the effect of granularity. In general it is not possible to determine which of the two opposing effects dominates. For the portfolio with a representative granularity in Table 11, both effects nearly cancel each other out, which is a rather encouraging result. Granularity in this portfolio is still conservatively measured since the underlying sample includes only the larger exposures of a bank. Therefore, the effect of granularity for this portfolio is arguably weaker than for portfolios of "average granularity" in real banks. This suggests that for such portfolios, PD heterogeneity would tend to overcompensate the granularity effect and EC^* and EC_{MFA} would provide conservative estimates. Further empirical work is warranted to confirm this indicative result.

Our analysis has shown that PD heterogeneity on the exposure level improves the performance of the analytic *EC* approximations relative to the situation of a granular portfolio with (only) sector-dependent PDs. The reason is that PD heterogeneity reduces the underestimation of *EC* that is caused by the granularity of the portfolio. This effect is even stronger if larger exposures or firms have lower PDs than smaller ones. Furthermore, PD heterogeneity appears not to affect the relative difference between EC_{MFA} and EC^*.

8. Summary and Conclusions

The minimum capital requirements for credit risk in the IRB approach of Basel II implicitly assume that banks' portfolios are well diversified across business sectors. Potential concentration risk in certain business sectors is covered by Pillar 2 of the Basel II Framework which comprises the supervisory review process.[30] To what extent the regulatory minimum capital requirements can understate economic capital is an empirical question. In this paper we provide a tentative answer by using data from the German central credit register. Credit risk is measured by economic capital in a multi-factor asset value model by Monte Carlo simulations.

In order to measure the impact of concentration risk on economic capital, we start in the empirical part with a benchmark portfolio that reflects average sector exposures of the German banking system. Since the exposure distributions across business sectors are similar in Belgium, France and Spain, we expect that our main results also hold for other European countries. Starting with the benchmark portfolio, we have successively increased sector concentration, considering degrees of sector concentration which are observable in real banks. The most concentrated portfolio contained exposures only to a single sector. Compared with the benchmark portfolio, economic capital for the concentrated portfolios can increase by almost 37% and be even higher in the case of a one-sector portfolio. This result clearly underlines the necessity to take inter-sector dependency into account for the measurement of credit risk. We subjected our results to various robustness checks. We found that the increase in economic capital may even be greater, contingent to the dependence structure.

Since concentration in business sectors can substantially increase economic capital, a tractable and robust calculation method for economic capital which avoids the use of computationally burdensome Monte Carlo simulations is desirable. For this purpose the theoretical part evaluates the accuracy of a model developed by Pykhtin (2004) which provides an analytical approximation of economic capital in a multi-factor framework. We have applied a simplified, more tractable version of the model which requires only sector-aggregates of exposure size, PD and expected loss severity. The dependence structure is captured by the correlation matrix of the original multi-factor model. Furthermore, we have evaluated the extent to which EC^*, as the first of two components in the analytic approximation of

[30] See BCBS (2004b), paragraphs 770-777.

economic capital, already provides a reasonable proxy of *EC*. EC^* refers to the economic capital for a single-factor model in which the sector-dependent asset correlations are defined by mapping the richer correlation structure of the multi-factor model. The benchmark for the approximation quality is always the *EC* of the original multi-factor model which is obtained from MC simulations.

We have shown that the analytic approximation formulae perform very well for portfolios with relatively granular and homogeneous sectors. This result holds for portfolios with different sector concentrations and for various factor weights and correlation assumptions. Furthermore, we have found that EC^* is relatively close to the simulation-based economic capital for most of the realistic input parameter tupels considered.

Finally, we explore the robustness of our results against the violation of two critical model assumptions, namely infinite granularity in every sector and sector-dependent PDs. We find that lower granularity and PD heterogeneity on the single exposure level have two counterbalancing effects on the performance of the analytic approximations for economic capital. The reduction of granularity induces the analytic approximation formulae to have a slight downward bias. In extreme cases of portfolios with the lowest granularity permissible by EU large exposure rules, the downward bias increases to 14%, depending on the sector structure of the portfolio.

Introducing heterogeneous PDs on the individual exposure level reduces economic capital, but does not affect the analytic approximations. As a consequence, the downward bias decreases. The relative error of the analytic approximation, measured relative to the simulation-based *EC* figure, lies in a range between –6% and +8%. In summary, we find that heterogeneity in individual PDs and low granularity partly balance each other in their impact on the performance of the analytic approximations. Which effect prevails depends on the specific input parameters. Indicative results suggest that in representative credit portfolios, PD heterogeneity will at least compensate for the granularity effect which suggests that the analytic formulae approximate *EC* reasonably well or err on the conservative side.

In the cases studied, it is possible to use the simplified version of the model which provides analytic approximations of *EC* without sacrificing much accuracy. This is an important result as it suggests that supervisors and banks can reasonably well approximate their economic capital for their credit portfolio by a relatively simple formula and without running computationally burdensome Monte Carlo simulations.

Further research seems to be warranted, particularly in further advancing Pykhtin's methodology in a direction which improves its approximation accuracy without extending data requirements. This could be achieved, for example, by exploring alternative ways to map the correlation matrix of the multi-factor model into sector-dependent asset correlations.

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Appendix A

Table 12: GICS Classification Scheme: Broad Sector and Industry Groups

A: Energy	A1: Energy
B: Materials	B1: Materials
C: Industrial	C1: Capital goods C2: Commercial Services and Supplies C3: Transportation
D: Consumer Discretionary	D1: Automobiles and Components D2: Consumer Durables and Apparel D3: Hotels, Restaurants and Leisure D4: Media D5: Retailing
E: Consumer Staples	E1: Food and Drug Retailing E2: Food, Beverage and Tobacco E3: Household and Personal Products
F: Health Care	F1: Health Care Equipment and Services F2: Pharmaceuticals and Biotechnology
G: Financials	G1: Banks G2: Diversified Financials G3: Insurance G4: Real estate
H: Information Technology	H1: Software and Services H2: Technology Hardware & Equipment H3: Semiconductors & Semiconductor Equipment
I: Telecommunication Services	I1: Telecommunication Services
J: Utilities	J1: Utilities

Table 13: Mapping NACE codes to GICS codes

2 (or more) -digit code	Description	Mapped to GICS
1	Agriculture and hunting	E
2	Forestry	B
5	Fishing	E
10	Coal mining	B
11	Crude petroleum and natural gas extraction	A
12	Mining of uranium and thorium	B
13	Mining of metal ores	B
14	Other mining and quarrying	B
15	Food and beverages manufacturing	E
16	Tobacco manufacturing	E
17	Textile manufacturing	D
18	Textile products manufacturing	D
19	Leather and leather products manufacturing	D
20	Wood products	D
21	Pulp, paper and paper products	B
22	Publishing and printing	C2

23	Manufacture of coke, refined petroleum products and nuclear fuel	A
24 (excl 244)	Chemicals and chemical products manufacturing	B
244	Pharmaceuticals	F
25	Rubber and plastic manufacturing	D
26	Other non-metallic mineral products	B
27	Basic metals manufacturing	B
28	Fabricated metal manufacturing	B
29	Machinery and equipment manufacturing	C1
30	Office machinery and computers manufacturing	H
31	Electrical machinery manufacturing	H
32	TV and communication equipment manufacturing	H
33	Medical and optical instruments manufacturing	F
34	Car manufacturing	D
35	Other transport equipment manufacturing	D
36	Furniture manufacturing	D
37	Recycling	J
40	Gas and electricity supply	J
41	Water supply	J
45	Construction	C1
50	Car sales, maintenance and repairs	D
51	Wholesale trade	C2
52 (excl 5211, 522,523)	Retail trade	D
522, 523	Consumer staples	E
55	Hotels and restaurants	D
60	Land transport	C3
61	Water transport	C3
62	Air transport	C3
63	Transport supporting activities and travel agencies	C3
64	Post and telecommunication	I
65	Financial institutions	G1
66	Insurance	G3
67	Support to financial institutions	G1
70	Real estate	G4
71	Machinery and equipment leasing manufacturing	C1
72	Computer and related activities	H
85	Health care and social work	F
90	Sewage and refuse disposal	J
96	Residential property management	G4

Table 14: Comparison of sector concentrations, aggregated exposure values over banks in Germany, France, Belgium and Spain

Sector	Germany	France	Belgium	Spain
A1: Energy	0.18%	0.88%	0.05%	1,05%
B1: Materials	6.01%	3.97%	7.45%	9,34%
C: Industrial[31]	52.36%	63.82%	54.77%	48,53%
C1: Capital Goods	11.53%		9.89%	32,90%
C2: Commercial Services and Supplies	33.69%		37.74%	10,20%
C3: Transportation	7.14%		7.14%	5,43%
D: Consumer Discretionary	14.97%	11.91%	15.77%	18,60%
E: Consumer Staples	6.48%	7.21%	7.05%	10,20%
F: Health Care	9.09%	5.00%	5.64%	1,85%
H1: Software and Services	3.20%	1.47%	1.86%	1,99%
I1: Telecommunication Services	1.04%	1.91%	0.54%	2,67%
J1: Utilities	6.67%	3.82%	6.87%	5,77%

Table 15: Correlation matrix based on MSCI EMU industry indices (based on weekly log return data covering the Nov 2002 - Nov 2003 period; in percentages).

	A	B	C1	C2	C3	D	E	F	H	I	J
A: Energy	100	62	66	43	62	67	78	70	50	47	72
B: Materials		100	91	78	77	85	73	69	74	68	69
C1: Capital Goods			100	76	80	92	74	68	81	72	75
C2: Comm. svs & supplies				100	66	81	58	53	71	58	52
C3: Transportation					100	78	68	59	70	65	64
D: Consumer discretionary						100	71	66	86	72	70
E: Consumer staples							100	75	62	60	70
F: Health Care								100	55	44	70
H: Information technology									100	69	58
I: Telecommu-nication services										100	67
J: Utilities											100

[31] Aggregate of C1, C2 and C3 only used for comparison with French data. Not used in the analysis.

Appendix B

The multi-factor adjustment Δt_q can be calculated according to Pykhtin (2004) as follows:

$$\text{(A1)} \qquad \Delta t_q = -\frac{1}{2l'(y)}\left[v'(y) - v(y)\left(\frac{l''(y)}{l'(y)} + y\right)\right]\Bigg|_{y=N^{-1}(1-q)}$$

where y denotes the single systematic risk factor.

The first and second derivatives of the loss distribution function in a one-factor model are

$$\text{(A2)} \qquad \begin{aligned} l'(y) &= \sum_{s=1}^{N} w_s \mu_s \hat{p}_s'(y) \\ l''(y) &= \sum_{s=1}^{N} w_s \mu_s \hat{p}_s''(y) \end{aligned}$$

where $\hat{p}_s'(y)$ and $\hat{p}_s''(y)$ are, respectively, the first and the second derivatives of the conditional probability of default.

$$\text{(A3)} \qquad \begin{aligned} \hat{p}_s'(y) &= -\frac{c_s}{\sqrt{1-c_s^2}} N'\left(\frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}}\right) \\ \hat{p}_s''(y) &= -\frac{c_s}{\sqrt{1-c_s^2}} \frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}} N'\left(\frac{N^{-1}(p_s) - c_s y}{\sqrt{1-c_s^2}}\right) \end{aligned}.$$

The factor loading in the ASRF* model is denoted by c_s which can be written as $c_s = r_s \bar{\rho}_s$ where $\bar{\rho}_s$ denotes the correlation between the composite sector factor Y_s and the systematic factor $\bar{Y}$ in the ASRF* model.

The conditional variance $v(y) \equiv \text{var}(L \mid \bar{Y} = y)$

$$
\begin{aligned}
v(y) = &\sum_{s=1}^{S}\sum_{t=1}^{S} w_s w_t \mu_s \mu_t \left[N_2\left(N^{-1}(\hat{p}_s(y)), N^{-1}(\hat{p}_t(y)), \rho_{st}^{Y}\right) - \hat{p}_s(y)\hat{p}_t(y) \right] \\
&+ \sum_{s=1}^{S} w_s^2 \mu_s^2 \left[\hat{p}_s(y) - N_2\left(N^{-1}(\hat{p}_s(y)), N^{-1}(\hat{p}_s(y)), \rho_{ss}^{Y}\right) \right] \\
v'(y) = &\sum_{s=1}^{S}\sum_{t=1}^{S} w_s w_t \mu_s \mu_t \hat{p}_s'(y) \left[N\left(\frac{N^{-1}(\hat{p}_t(y)) - \rho_{st}^{Y} \hat{p}_s(y)}{\sqrt{1-\left(\rho_{st}^{Y}\right)^2}} \right) - \hat{p}_t(y) \right] \\
&+ \sum_{s=1}^{S} w_s^2 \mu_s^2 \hat{p}_s'(y) \left[1 - 2 \cdot N\left(\sqrt{\frac{1-r_s^2}{1+r_s^2}} N^{-1}(\hat{p}_s(y)) \right) \right]
\end{aligned}
$$

where $N_2(\ )$ denotes the cumulative distribution function of the bivariate-normal distribution and ρ_{st}^{Y} has the meaning of a conditional asset correlation for two exposures in sectors t and s, conditional on $\bar{Y}$. This conditional asset correlation can be written as

$$
r_{st}^{Y} = \frac{r_s r_t r_{st} - c_s c_t}{\sqrt{\left(1 - c_s^2\right)\left(1 - c_t^2\right)}} .
$$

Appendix C

In Pykhtin (2004) the coefficients $b_1,\ldots,b_S$ are obtained by maximizing the correlation between $\bar{Y}$ and the risk factors $Y_{1,\ldots,}Y_S$ which leads to the following optimization problem:

$$\max_{b_1,\ldots,b_S} \sum_{s=1}^{S} \gamma_s \sum_{j=1}^{S} \alpha_{s,j} b_j \ .$$

subject to $\sum_{s=1}^{S} b_s^2 = 1$. The solution of this optimization problem is given by

$$b_j = \sum_{s=1}^{S} \frac{\gamma_s}{\lambda} \alpha_{s,j} \ .$$

λ is the Lagrange multiplier chosen to satisfy the constraint. Again there is no unique solution for γ_s. We follow Pykhtin who reported good results when defining

$$\gamma_s = w_s \mu_s N\left(\frac{N^{-1}(p_s) - r_s N^{-1}(q)}{\sqrt{1-r_s^2}}\right).$$

Appendix D

Table 16: Descriptive statistics of exposure distribution of a portfolio of 11 sectors, representative in terms of granularity

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	1	NA	1.2	NA	1.2
2	36	0	1.3	4.7	8.9	43.2
3	69	0	2.2	6.1	12.8	127.6
4	203	0	1.7	5.3	10.5	152.3
5	43	0.1	2.1	5.4	10.5	60.0
6	90	0	1.4	5.1	9.3	112.2
7	39	0.1	1.3	4.9	10.0	42.2
8	55	0.2	1.8	4.8	11.3	74.2
9	19	0.1	0.7	3.6	5.8	22.0
10	6	0.2	0.6	3.0	7.8	8.5
11	40	0.0	1.3	5.8	11.6	68.8

Table 17: Descriptive statistics of exposure distribution of a low granular portfolio of 11 sectors

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	11	NA	11	NA	11
2	8	32	47	47	47	47
3	6	92	120	120	120	120
4	17	100	120	120	120	120
5	10	6	47	47	47	47
6	8	58	120	120	120	120
7	9	13	47	47	47	47
8	9	33	47	47	47	120
9	5	4	47	47	47	47
10	2	16	16	31.5	47	47
11	9	24	47	47	47	47

Table 18: Scaled PD distribution of a portfolio of 11 sectors, representative in terms of granularity

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	1.0%	NA	1.0%	NA	1.0%
2	36	0.0%	0.5%	1.0%	2.0%	8.8%
3	69	0.1%	0.7%	1.4%	2.3%	8.9%
4	203	0.0%	0.9%	1.6%	3.0%	14.4%
5	43	0.1%	0.6%	1.4%	2.6%	5.7%
6	90	0.0%	0.7%	1.2%	2.5%	12.7%
7	39	0.0%	0.8%	1.9%	3.4%	5.1%
8	55	0.0%	0.3%	0.6%	1.3%	8.4%
9	19	0.0%	0.5%	1.6%	2.5%	5.5%
10	6	0.3%	2.4%	4.4%	5.7%	7.8%
11	40	0.0%	0.2%	3.4%	0.6%	2.4%

Table 19: Scaled PD distribution of a low granular portfolio of 11 sector

Sector	Exposure No.	Minimum	25% percentile	Median	75% percentile	Maximum
1	1	1.0%	NA	1.0%	NA	1.0%
2	8	0.3%	1.3%	1.3%	1.3%	4.2%
3	6	0.4%	1.5%	1.5%	1.5%	5.1%
4	17	0.1%	1.8%	1.8%	1.8%	5.9%
5	10	0.1%	1.5%	1.5%	1.5%	4.9%
6	8	0.4%	1.4%	1.4%	1.4%	4.7%
7	9	0.1%	1.7%	1.7%	1.7%	5.8%
8	9	0.1%	0.7%	0.7%	0.7%	2.4%
9	5	0.3%	1.2%	1.2%	1.2%	3.9%
10	2	2.4%	2.4%	2.4%	2.4%	2.4%
11	9	0%	0.3%	0.3%	0.3%	1%

Table 20: Quality distribution of German firms in the Bundesbank database

Rating grade	AAA	AA	A	BBB	BB	B
Share in percent	2	6	11	55	24	2
(unscaled) PD in percent	0.01	0.02	0.07	0.26	0.87	3.27